Nonlinear Unsaturated Zone Behaviour in Groundwater Time Series Analysis

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Introduction

Time series analysis is becoming a popular method to analyse groundwater table measurements in an observation well. The Python package Pastas applies time series analysis with transfer function noise (TFN) modelling using predefined response functions. To do this successfully the response function of each stress has to be estimated, including the uncertainty [1]. The most commonly used stress to model groundwater table fluctuations is the recharge flux. Usually the recharge flux is approximated by a linear combination of precipitation and potential evaporation. Recently, a nonlinear recharge model was added to Pastas to account for the nonlinearity of the unsaturated zone [2]. **The goal of this study was to see if the nonlinear recharge model improves the TFN model's ability in predicting the groundwater table, influenced by unsaturated zone behaviour**.

Nonlinear Behaviour of the Unsaturated Zone

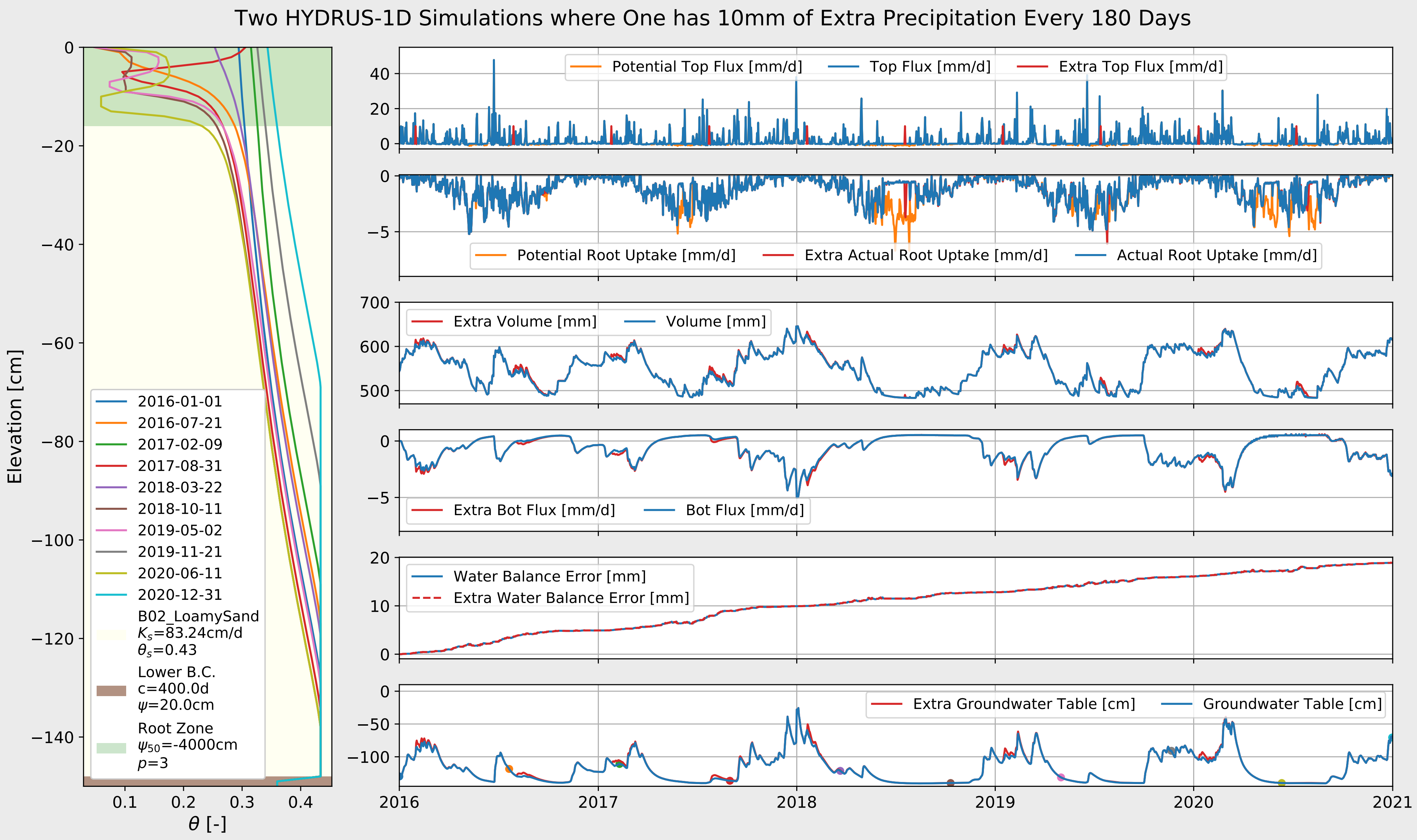

Fig. 1: HYDRUS-1D Simulations for a Homogeneous Profile Created with pHydrus [3]

With a linearly behaving unsaturated zone, one would expect that the groundwater response to the same recharge event is equivalent. Similarly, if the recharge flux is twice as large, the response of the groundwater table should also be twice as large. HYDRUS-1D solves Richards' equation for variably saturated flow, which accounts for the nonlinear relation between degree of saturation and the hydraulic conductivity. As a result the recharge to the groundwater table, through the unsaturated zone, is highly nonlinear. Another source of nonlinearity is the actual evaporation, which is dependent on the degree of saturation of the root zone [4].

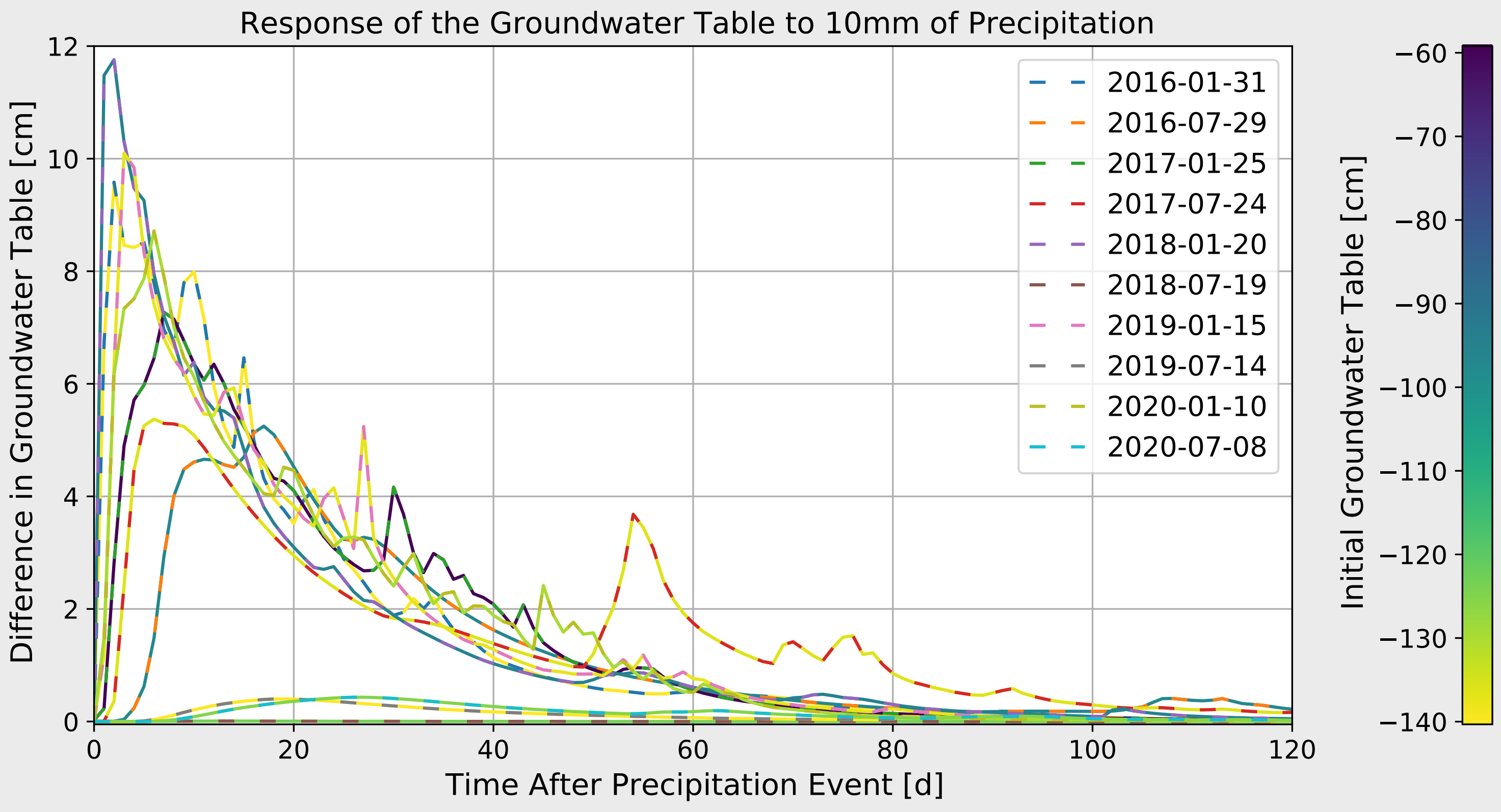

Fig. 2: Difference in Groundwater Table Response Due To 10mm Extra Precipitation

In Figure 1 two HYDRUS-1D simulations, created with pHydrus, are shown [3]. The first simulation (blue) uses normal weather data. The second simulation (red) uses the same weather data but adds 10mm of extra precipitation every 180 days. Figure 2 shows the response to the 10mm precipitation event via the difference in groundwater table between the two simulations. Here it is seen that the same precipitation event can have a varying effect on the groundwater table. In some cases the there is no response at all (brown dashes 2018-07-19). In other cases there are are multiple peaks, even up to 55 days after the event (red dashes 2017-07-24).

Generating Synthetic Time Series

To test the performance of the nonlinear recharge model, synthetic time series for the groundwater table are generated with HYDRUS-1D. In total thirty-five different scenarios are simulated with a bottom boundary condition that is as realistic as possible. The models are varied between five homogeneous soil profiles. The other variable is the unsaturated zone thickness (denoted as the drainage level), which varies in between -100 and -500 cm. The *van Genuchten* soil hydraulic model with an air-entry value of -2cm is used with parameters retrieved from Dutch soils in the Staring series [5]. The saturated hydraulic conductivities for these soils range between 83.24 and 0.90 cm/d. The time series are created with a daily time step for 22 years with KNMI weather data from De Bilt.

Transfer Function Noise Modelling with Pastas

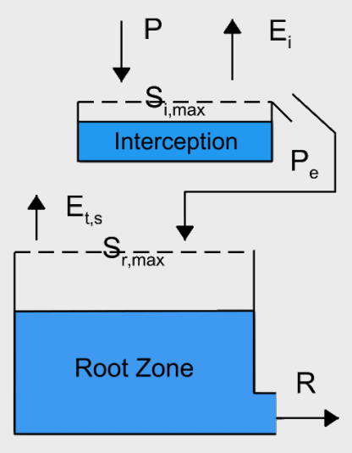

Fig. 3: Nonlinear Recharge Model

TFN models use a recharge stress and a block response function to estimate the contribution of precipitation and evaporation to the groundwater table fluctuations. The recharge flux can be estimated using the linear or nonlinear recharge model. The linear recharge model estimates the recharge as $R = P - fE_p$. The nonlinear recharge model uses a bucket model approach as seen in Fig. 3. The TFN model estimates the response of the groundwater table to the recharge. Figure 4 shows the two most commonly used impulse response functions, the exponential and gamma response function.

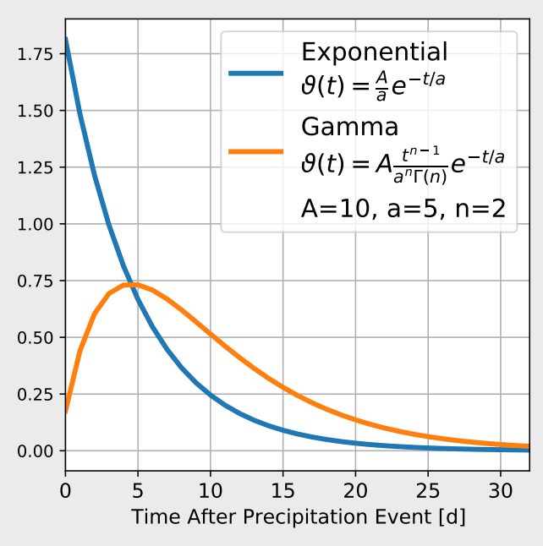

Fig. 4: Block Response

Linear versus Nonlinear Recharge Model

The synthetic groundwater time series generated with HYDRUS-1D are modelled with different TFN models, varying between the linear & nonlinear recharge model and the response functions. The period 2002-2017 is used for calibration and 2018-2020 is used for validation. As a goodness-of-fit metric, the R^2 is used to indicate the proportion of improvement due to the TFN model.

- As seen in Figures 5 and 6, for all synthetic groundwater table time series the nonlinear recharge model (orange) improves the TFN simulation compared to the linear recharge model (blue);
- The nonlinear recharge model, in combination with the exponential response function, gives reasonable estimates of the recharge flux and evaporation reduction;
- Especially in dry summers, the nonlinear recharge model shows a better fit than the linear recharge model. The linear recharge model generally undershoots the groundwater table in summer;
- The fit of the linear recharge model improves with decreasing unsaturated zone thickness. The use of the gamma response function, which accounts for dispersion in the unsaturated zone, also improves the fit (blue line Fig. 6).

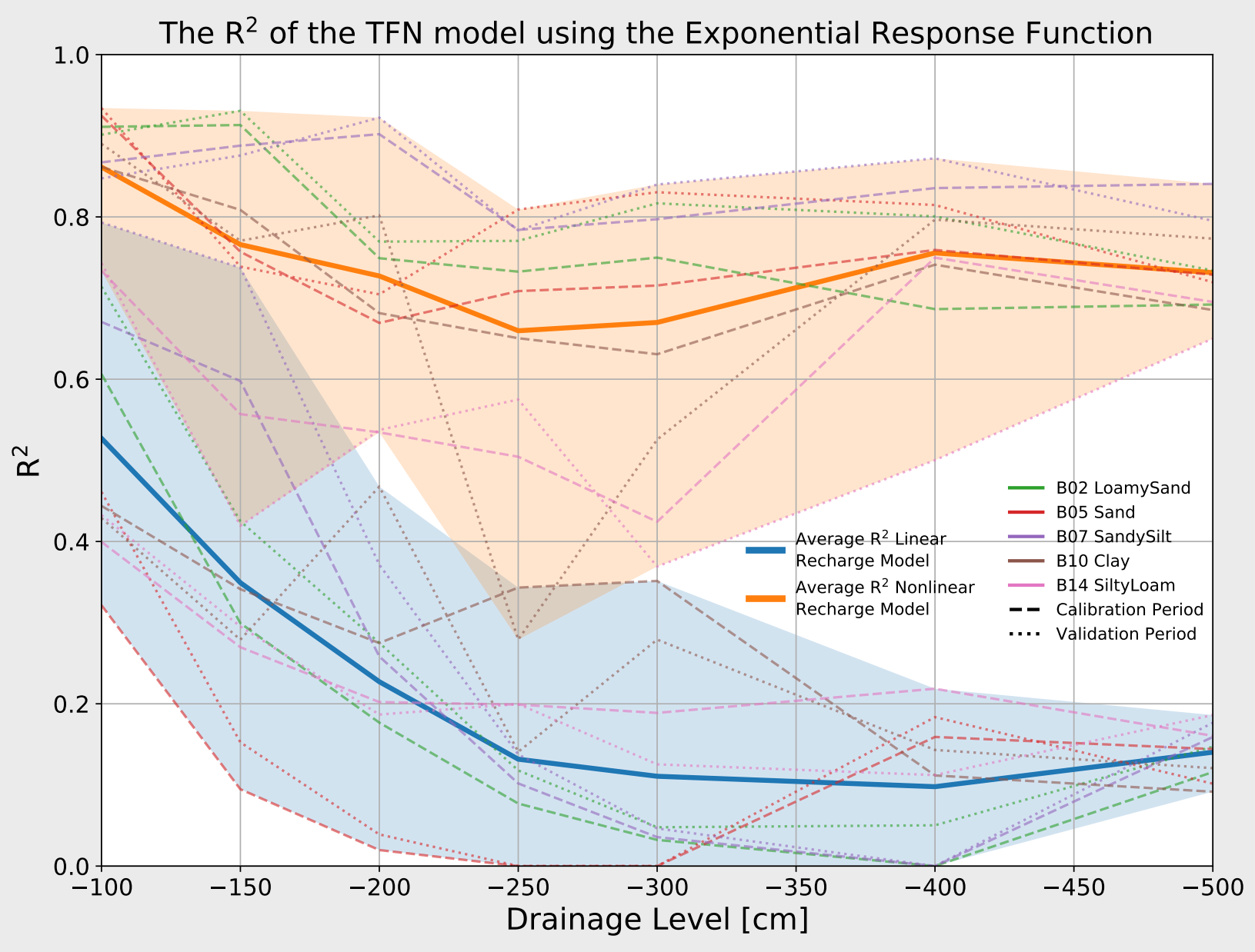

Fig. 5: TFN Results with Exponential Response Function

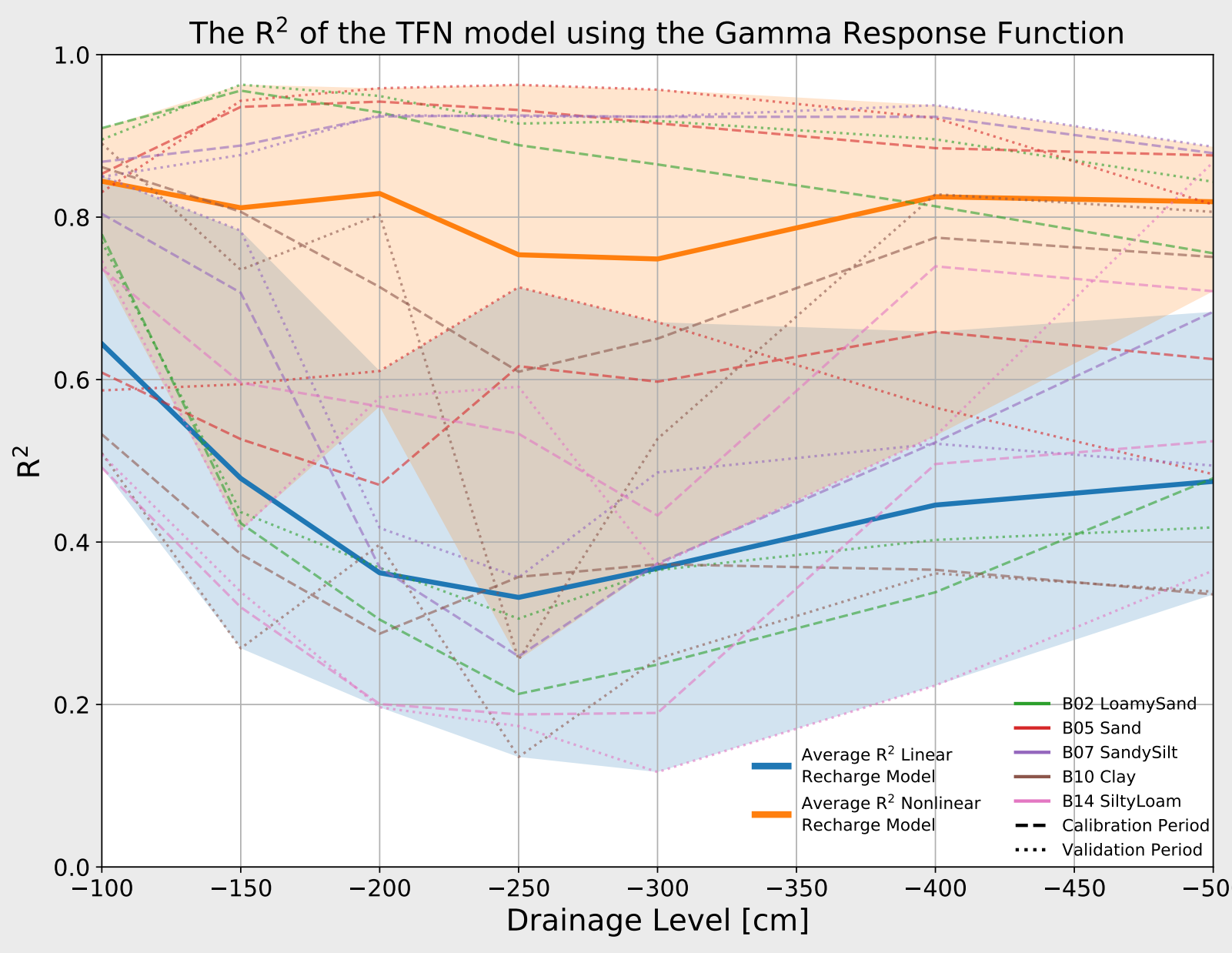

Fig. 6: TFN Results with Gamma Response Function

Discussion

- To what extent are the results from this study transferable to observed groundwater table time series?
- Generating synthetic time series is a challenging task. However, is it useful to benchmark these time series against measured time series or existing models?
- How to incorporate upwards recharge in the nonlinear recharge model?

References

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